ON THE THEORY OF OPTIMUM LAND REFORM

BY

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SUMMARY

Recently in an interesting paper on optimum land reform, Raj Krishna has dealt with theoretical explanations for the existence of some of the stylized facts of agriculture in developing countries with the help of a Cobb-Douglas production fnnction model. In this paper our objective is to show that Raj Krishna's results are equally valid for more general production functions which do not assume unitary elasticity of substitution between [land and labor functions as Cobb-Douglas does. Specifically, we will deal with constant elasticity of substitution (hereafter referred to as CES) and variable elasticity of substitution (hereafter referred to as VES) production functions.

1. Recently in an interesting paper on optimum land reform, Raj Krishna has dealt with theoretical explanations for the existence of some of the stylized facts of agriculture in developing countries with the help of a Cobb-Douglas production model. By characterizing the dualistic nature of agriculture partially in terms of differential wages for small and large farms, he derived the following results under a Cobb-Douglas technology with higher wage rates for labor on large farms than on small farms :

(*i*) Higher output per hectare on small farms.

(*ii*) Higher employment per hectare on small farms.

(*iii*) Higher productivity per work unit (labor productivity) on large farms.

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(*iv*) Finally defining total output and total employment by adding optimum outputs and employment on small and large farms. respectively with given land distribution, Raj Krishna showed how land redistribution in favor of small farms will lead to higher total output and employment in the economy.

In this paper our objective is to show that Raj Krishna's results are equally valid for more general production functions which do not assume unitary elasticity of substitution between land and labor functions as Cobb-Douglas does. Specifically, we will deal with constant elasticity of substitution (hereafter referred to as CES) and variable elasticity of substitution (hereafter referred to as VES) poduction functions.

II. The Short-run Two Farm Model with CES and VES Functions. Following Raj Krishna, we will divide the whole farm sector into small farms and large farms and denote the division of given land area (L) between small and large farms by ratios (x) and (l-x) respectively.

A. With CES Function. Now, given a CES production function

$$Q=[d(xL)^{-\rho}+(l\text{-}d)N^{-\rho}]^{-\frac{1}{\rho}} \qquad \ldots(1)$$

where Q is output, L is the fixed land area, and N is employment, the opti mum employment on any farm with given wage (w) will be : 1

$$N=xL\left[\frac{d}{\left(\frac{1-d}{w}\right)^{\frac{\rho}{1+\rho}}-(1-d)}\right]^{-\frac{1}{\rho}} \qquad \ldots(2)$$

Since, xL is fixed and output can be increased only by increasing the level of N, (assuming the price of output to be unity) the optimum output on any farm will be :2

$$Q=xL\left[d+(1-d)\frac{d}{\left(\frac{1-d}{w}\right)^{\frac{\rho}{1+\rho}}-(1-d)}\right]^{-\frac{1}{\rho}} \qquad \ldots(3)$$

1(2) is obtained by differentiating (1) with respect to N and setting it equal to w and solving the resulting equation for N.

2(3) is obtained by substituting (2) for N in (1) and simplifying the resulting equation.

Thus, output per hectare will be :

$$q=\left[d+(1-d)\frac{d}{\left(\frac{1-d}{w}\right)^{\frac{\rho}{1+\rho}}-(1-d)}\right]^{-\frac{1}{\rho}} \qquad \ldots(4)$$

Employment per hectare will be :

$$n=\left[\frac{d}{\left(\frac{1-d}{w}\right)^{\frac{\rho}{1+\rho}}-(1-d)}\right]^{-\frac{1}{\rho}} \qquad \ldots(5)$$

and productivity per work-unit will be :

$$\rho=\frac{\left[d(xL)^{-\rho}+(1-d)(xL)^{-\rho}\frac{d}{\left(\frac{1-d}{w}\right)^{\frac{\rho}{1+\rho}}-(1-d)}\right]^{\frac{1}{\rho}}}{(xL)\left[\frac{d}{\left(\frac{1-d}{w}\right)^{\frac{\rho}{1+\rho}}-(1-d)}\right]^{-\frac{1}{\rho}}} \qquad \ldots(6)$$

$$=\left(\frac{w}{1-d}\right)^{\frac{1}{1+\rho}}$$

It could be seen from the above equations (4 to 6) that, as under Cobb-Douglas, under CES production function also with partial dualism in the labor market (represented by a higher wage (w_2) on large farms compared to a lower wage (w_1) on small farms), the small farms will have higher levels of per hectare output and per hectare employment, and lower productivity per work-unit as compared to larger farms. In other words :

$$\left[d+(1-d)\frac{d}{\left(\frac{1-d}{w_1}\right)^{\frac{\rho}{1+\rho}}-(1-d)}\right]^{-\frac{1}{\rho}}$$

$$>\left[d+(1-d)\frac{d}{\left(\frac{1-d}{w_2}\right)^{\frac{\rho}{\rho+1}}-(1-d)}\right]^{-\frac{1}{\rho}} \qquad \ldots(7)$$

$$\left[\frac{d}{\left(\frac{1-d}{w_1}\right)^{\frac{\rho}{1+\rho}}-(1-d)}\right]^{-\frac{1}{\rho}}>\left[\frac{d}{\left(\frac{1-d}{w_2}\right)^{\frac{\rho}{1+\rho}}-(1-d)}\right]^{-\frac{1}{\rho}} \qquad \ldots(8)$$

and

$$\left(\frac{w_1}{1-d}\right)^{\frac{1}{1+\rho}}<\left(\frac{w_2}{1-d}\right)^{\frac{1}{1+\rho}} \qquad \ldots(9)$$

for all values of ρ between -1 and $+\infty$.

Again following Raj Krishna let us define the total output in the economy as:

$$Q_T=Q_S+Q_L \qquad \ldots(10)$$

where Q_T is total output, Q_S is small farms output, and Q_L is large farms output. Since land is divided between the two farms as x and $(1-x)$,

$$Q_T=xL\left[d+(1-d)\frac{d}{\left(\frac{1-d}{w_1}\right)^{\frac{\rho}{1+\rho}}-(1-d)}\right]^{-\frac{1}{\rho}}$$

$$+(1-x)L\left[d+(1-d)\frac{d}{\left(\frac{1-d}{w_2}\right)^{\frac{\rho}{1+\rho}}-(1-d)}\right]^{-\frac{1}{\rho}} \qquad \ldots(11)$$

so that

$$\frac{dQ_T}{dx}=L\left[\left\{d+(1-d)\frac{d}{\left(\frac{1-d}{w_1}\right)^{\frac{\rho}{1+\rho}}-(1-d)}\right\}^{-\frac{1}{\rho}}\right.$$

$$\left.-\left\{d+(1-d)\frac{d}{\left(\frac{1-d}{w_2}\right)^{\frac{\rho}{1+\rho}}-(1-d)}\right\}^{-\frac{1}{\rho}}\right]>0 \qquad \ldots(12)$$

since $w_2>w_1$.

Similarly we can also show that land redistribution in favor of small farms will increase total employment. Defining total employment as

$$N_T=N_S+N_L \qquad \ldots(13)$$

Where N_T is total employment, N_S is employment on small farms and N_L is employment on large farms, so that

$$N_T=(xL)\left[\frac{d}{\left(\frac{1-d}{w_1}\right)^{\frac{\rho}{1+\rho}}-(1-d)}\right]^{-\frac{1}{\rho}}$$

$$+(1-x)L\left[\frac{d}{\left(\frac{1-d}{w_2}\right)^{\frac{\rho}{1+\rho}}-(1-d)}\right]^{-\frac{1}{\rho}} \quad \ldots(14)$$

and

$$\frac{dN_T}{dx}=L\left[\left\{\frac{d}{\left(\frac{1-d}{w_1}\right)^{\frac{\rho}{1+\rho}}-(1-d)}\right\}^{-\frac{1}{\rho}}\right.$$

$$\left.-\left\{\frac{d}{\left(\frac{1-d}{w_2}\right)^{\frac{\rho}{1+\rho}}-(1-d)}\right\}^{-\frac{1}{\rho}}\right]>0 \quad (15)$$

since $w_2>w_1$

Thus under *CES* also we get the same result that redistribution of land in favor of small farms will increase both employment and output in the economy.

B. With VES Function

Given the *VES* production function[3]

$$Q=b(xL)^{a(1-d\rho)}\,[N+(\rho-1)\,xL]^{ad\rho} \quad \ldots(16)$$

with the following restrictions

$$b>0;\ a>0:\ 0<d<1;\ 0\leqslant d\rho\leqslant 1 \text{ and}$$

$$\frac{N}{xL}>\left(\frac{1-\rho}{1-d\rho}\right),$$

with constant returns to scale, $a=1$, (16) reduces to

$$Q=b(xL)^{1-d\rho}\,[N+(\rho-1)\,xL]^{d\rho}. \quad \ldots(17)$$

[3]Though there are different variants of the *VES* function, for our purposes we use Revankar's [2].

Given (17), the optimum level of employment on any farm at the given wage (w) will be[4] :

$$N = xL\left(\frac{w}{d\rho b}\right)^{\frac{1}{d_\rho - 1}}(2-\rho) \quad \text{.. (18)}$$

optimum output will be :

$$Q = b(xL)\left[\left(\frac{w}{d\rho b}\right)^{\frac{1}{d_\rho - 1}}(2-\rho)+(\rho-1)\right]^{d\rho} \quad \text{...(19)}$$

output per hectare will be :

$$q = b\left[\left(\frac{w}{d\rho b}\right)^{\frac{1}{d_\rho - 1}}(2-\rho)+(\rho-1)\right]^{d\rho} \quad \text{...(20)}$$

Employment per hectare will be :

$$n = \left(\frac{w}{d\rho b}\right)^{\frac{1}{d_\rho - 1}}(2-\rho) \quad \text{...(21)}$$

and productivity per work unit will be :

$$p = \frac{b\left[\left(\frac{w}{d\rho b}\right)^{\frac{1}{d_\rho - 1}}(2-\rho)+(\rho-1)\right]^{d\rho}}{\left(\frac{w}{d\rho b}\right)^{\frac{1}{d_\rho - 1}}(2-\rho)} \quad \text{...(22)}$$

again it could be easily seen that the results that were obtained under Cobb-Douglas and *CES* production functions hold true under *VES* functions also.

As earlier, defining total output (Q_T) as ($Q_S + Q_L$),

$$Q_T = b(xL)\left[\left(\frac{w_1}{d\rho b}\right)^{\frac{1}{d_\rho - 1}}(2-\rho)+(\rho-1)\right]^{d\rho} + b(1-x)L\left[\left(\frac{w_2}{d\rho b}\right)^{\frac{1}{d_\rho - 1}}(2-\rho)+(\rho-1\right]^{d\rho} \quad (23)$$

[4] Since

$$\frac{dQ}{dN} = d\rho b\,(xL)^{1-d\rho}\,[N+(\rho-1)L]^{d\rho-1} = w$$

hence (18).

so that

$$\frac{dQ_T}{dx}=bL\Big[\Big\{\Big(\frac{w_1}{d\rho b}\Big)^{\frac{1}{d\rho-1}}(2-\rho)+(\rho-1)\Big\}^{d\rho}$$

$$-\Big\{\Big(\frac{w_2}{d\rho b}\Big)^{\frac{1}{d\rho-1}}(2-\rho)+(\rho-1)\Big\}^{d\rho}\Big]>0 \qquad ...(24)$$

since $w_2>w_1$ and $(d\rho-1)<0$.

Similarly as earlier, defining total employment N_T as (N_S+N_L)

$$N_T=(xL)\Big(\frac{w_1}{d\rho b}\Big)^{\frac{1}{d\rho-1}}(2-\rho)$$

$$+(1-x)L\Big(\frac{w_2}{d\rho b}\Big)^{\frac{1}{d\rho-1}}(2-\rho) \qquad ...(25)$$

so that

$$\frac{dN_T}{dx}=L\Big(\frac{w_1}{d\rho b}\Big)^{\frac{1}{d\rho-1}}(2-1)-L\Big(\frac{w_2}{d\rho b}\Big)^{\frac{1}{d\rho-1}}(2-\rho)>0 \quad ...(26)$$

since $w_2>w_1$ and $(d\rho-1)<0$.

From (24) and (26) it is clear that land redistribution in favor of small farms under *VES* production functions also will lead to an increase in total output and employment in the economy.

Thus, to conclude, Raj Krishna's general approach of analyzing the optimum land reform under realistic nature of agricultural economies seems to lead to consistent results and conclusions even under more general production fuhctions This shows the robustness of Raj Krishna's overall approach.

REFERENCES

[1] Raj Krishna (1977) : "Toward a theory of optimum land reform for a dualistic agriculture, *J. Ind. Soc. Agri. Stati.* 1977.

[2] Revankar. N.S., (1968) : "A class of variable elasticity of substitution production functions," *Econometrica* 39: 61-71, 1968.